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Estimating the subcellular absorption of electric field energy: equations for an ellipsoidal single shell model

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Abstract

An oriented single shell model is used to describe the absorption of electric field energy for a cell of the general ellipsoidal shape exposed to a homogeneous AC-field. A finite element approach allowed us to derive characteristic equations describing the dependence of the field distribution on the cell geometry, the electric properties of the structural media, membrane and bulk solutions, as well as on the field frequency with a subcellular resolution. Finally, equations were derived for the absorption at certain sites of the model. The model allows for the introduction of frequency-dependent cellular media properties. Experimentally, the new cell parameters can be verified by dielectric single-cell spectroscopy.

Keywords: Power dissipation; Absorption; Finite element model; Dielectric dispersion; Electro-smog

1. Introduction

1.1. Absorption of a homogenous volume element

The Grotthus–Draper principle implies that only the absorbed part of the electromagnetic field energy may influence an exposed object. Therefore, the magnetic cell properties can be neglected. For a small volume element of homogeneous electrical properties, length l and cross-sectional area A, the current depends on the applied field strength and the impedance Z^*:

$$Z^* = \frac{1}{\sigma + j\omega\varepsilon\varepsilon_0} \quad (1)$$

where σ, j, ω and $\varepsilon\varepsilon_0$ are the specific conductivity, imaginary unit the circular frequency and the permittivity, respectively. The asterisk (*) denotes the complex properties of the impedance. The volume-specific absorption P_V of a volume element is caused by Ohmic heating:

$$P_V = E_{eff}^2 \sigma \quad (2)$$

where E_{eff} is the effective local field strength.

1.2. Model for the local field

To describe structural dispersions we have applied our cell polarization model adapted to ellipsoidal cells [1,2]. It consists of a homogeneous cytoplasm covered by a homogeneous membrane layer of constant thickness. The model provides access to the potentials, the local fields and thus the local absorption in the cellular media.

2. Results and discussion

2.1. Estimating the local fields

Using the voltage divider properties of the model [1], we derived equations for the absorption at certain sites within (cyt) and in the vicinity (ext1, ext2, mem1, mem2) of the cell (Fig. 1, lower left).

The local field in a homogeneous object of ellipsoidal shape, which is exposed to a homogeneous field, is constant [3]. This condition applies to the cytoplasm. The field strength at the cell's surface (ext) can be found from the boundary conditions for potentials and fields at the cell surface and the effective properties of the Maxwellian equivalent body of the cell [1]. Assuming isotropic membrane properties the field in the central plane of the membrane at mem2 is the average of the internal and the

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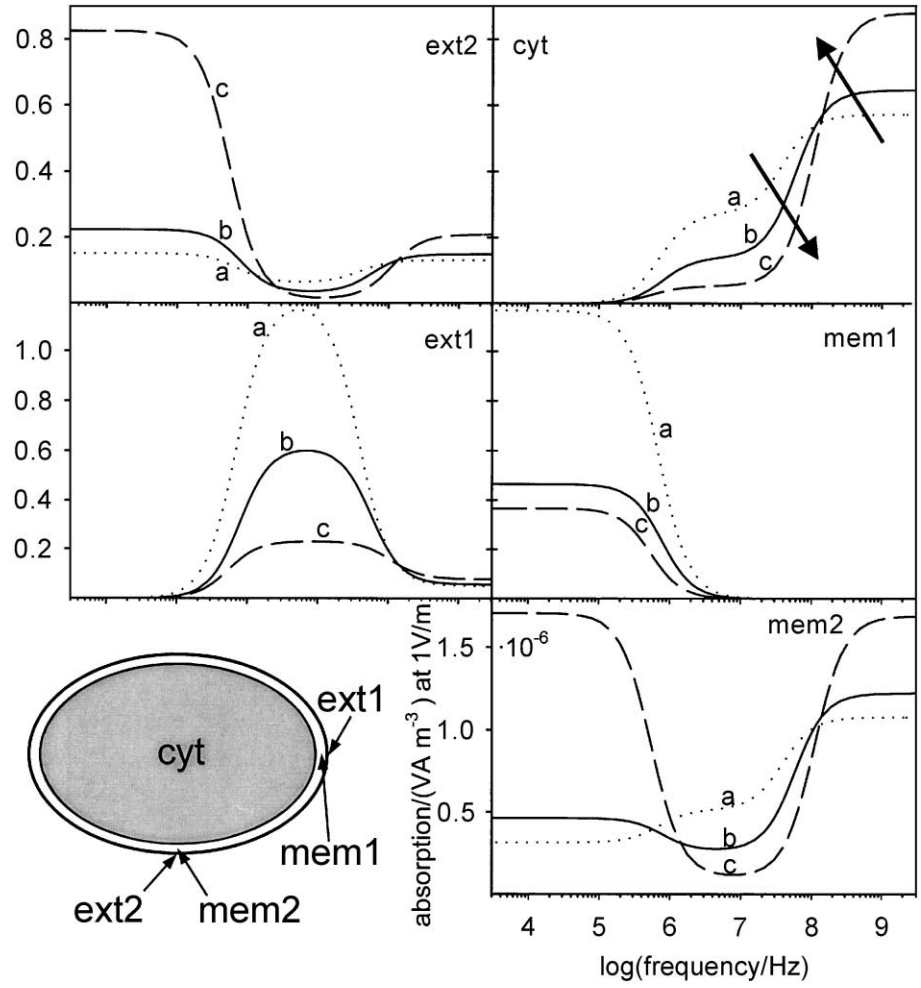

Fig. 1. Frequency spectra of absorption at the sites defined in this figure for different axes (*a*, *b* or *c*) oriented in parallel to the field. Lower left*:* Definition of certain sites of the cell model: inside the cytoplasm (cyt), inside the membrane (polar site: mem1, equatorial site: mem2), and in the external medium at the cell surface (polar site: ext1, equatorial site: ext2), respectively. ext1, ext2: The frequency dependence of absorption reflects the distortion of the external field by the polarization of the cell. cyt: Absorption in the cytoplasm. Please note that cell orientation changes the absorption in the medium and high frequency ranges in different ways (arrows). For high frequencies the local absorption within the cytoplasm exceeds that at all other sites. mem1, mem2: The polar membrane absorption (mem1) vanishes around 10 MHz. Please note that the absorption at mem2 is very small for all frequencies.

external fields at the surfaces. The local field at mem1 may reach extremely high values. It is oriented in parallel to the external field. The following equations can directly be derived from the electrical model:

$$E_{\mathrm{cyt}} = \frac{Z_{\mathrm{i}}^{*} E r_{\mathrm{inf}}}{(Z_{\mathrm{i}}^{*} + Z_{\mathrm{m}}^{*} + Z_{\mathrm{e}}^{*}) r} \tag{3}$$

$$E_{\mathrm{ext1}} = \frac{Z_{\mathrm{e}}^{*} E r_{\mathrm{inf}}}{(Z_{\mathrm{i}}^{*} + Z_{\mathrm{m}}^{*} + Z_{\mathrm{e}}^{*})(r_{\mathrm{inf}} - r)} \tag{4}$$

$$E_{\mathrm{ext2}} = \frac{(Z_{\mathrm{i}}^{*} + Z_{\mathrm{m}}^{*}) E r_{\mathrm{inf}}}{(Z_{\mathrm{i}}^{*} + Z_{\mathrm{m}}^{*} + Z_{\mathrm{e}}^{*}) r} \tag{5}$$

$$E_{\mathrm{mem1}} = \frac{Z_{\mathrm{m}}^{*} E r_{\mathrm{inf}}}{(Z_{\mathrm{i}}^{*} + Z_{\mathrm{m}}^{*} + Z_{\mathrm{e}}^{*}) d} \tag{6}$$

$$E_{\mathrm{mem2}} = \frac{E_{\mathrm{cyt}} + E_{\mathrm{ext2}}}{2} = \frac{(2Z_{\mathrm{i}}^{*} + Z_{\mathrm{m}}^{*}) E r_{\mathrm{inf}}}{2(Z_{\mathrm{i}}^{*} + Z_{\mathrm{m}}^{*} + Z_{\mathrm{e}}^{*}) r} \tag{7}$$

r_{inf} is the influential radius; r is the semi axis oriented in field direction (see Ref. [2]). Indices i, m, and e stand for cytoplasm, membrane and external medium, respectively.

Table 1
Simple equations to estimate the field for the different sites of the cells for E_{mem1} please see also [4]

Frequency range	Very low	Medium	Beyound Maxwell–Wagner-dispersion of bulk media
E_{cyt}	≈ 0	$\approx \frac{\sigma_{\mathrm{e}} E r_{\mathrm{inf}}}{r(\sigma_{\mathrm{e}} - \sigma_{\mathrm{i}}) + r_{\mathrm{inf}} \sigma_{\mathrm{i}}}$	$\approx \frac{\varepsilon_{\mathrm{e}} E r_{\mathrm{inf}}}{r(\varepsilon_{\mathrm{e}} - \varepsilon_{\mathrm{i}}) + r_{\mathrm{inf}} \varepsilon_{\mathrm{i}}}$
E_{ext1}	≈ 0	$\approx \frac{\sigma_{\mathrm{i}} E (r_{\mathrm{inf}} - r)}{r(\sigma_{\mathrm{e}} - \sigma_{\mathrm{i}}) + r_{\mathrm{inf}} \sigma_{\mathrm{i}}}$	$\approx \frac{\varepsilon_{\mathrm{i}} E (r_{\mathrm{inf}} - r)}{r(\varepsilon_{\mathrm{e}} - \varepsilon_{\mathrm{i}}) + r_{\mathrm{inf}} \varepsilon_{\mathrm{i}}}$
E_{ext2}	$\approx \frac{E r_{\mathrm{inf}}}{r}$	$\approx \frac{\sigma_{\mathrm{e}} E r_{\mathrm{inf}}}{r(\sigma_{\mathrm{e}} - \sigma_{\mathrm{i}}) + r_{\mathrm{inf}} \sigma_{\mathrm{i}}}$	$\approx \frac{\varepsilon_{\mathrm{e}} E r_{\mathrm{inf}}}{r(\varepsilon_{\mathrm{e}} - \varepsilon_{\mathrm{i}}) + r_{\mathrm{inf}} \varepsilon_{\mathrm{i}}}$
E_{mem1}	$\approx \frac{E r_{\mathrm{inf}}}{d}$	$\approx \frac{\varepsilon_{\mathrm{e}} E r_{\mathrm{inf}}}{(r_{\mathrm{inf}} - r)\varepsilon_{m} + d\varepsilon_{\mathrm{e}}}$	$\approx \frac{\varepsilon_{\mathrm{i}} \varepsilon_{\mathrm{e}} E r_{\mathrm{inf}}}{\varepsilon_{\mathrm{m}}(r(\varepsilon_{\mathrm{e}} - \varepsilon_{\mathrm{i}}) + r_{\mathrm{inf}} \varepsilon_{\mathrm{i}})}$
E_{mem2}	$\approx \frac{E r_{\mathrm{inf}}}{2r}$	$\approx \frac{\sigma_{\mathrm{e}} E r_{\mathrm{inf}}}{r(\sigma_{\mathrm{e}} - \sigma_{\mathrm{i}}) + r_{\mathrm{inf}} \sigma_{\mathrm{i}}}$	$\approx \frac{\varepsilon_{\mathrm{e}} E r_{\mathrm{inf}}}{r(\varepsilon_{\mathrm{e}} - \varepsilon_{\mathrm{i}}) + r_{\mathrm{inf}} \varepsilon_{\mathrm{i}}}$

Table 2
Parameters for model calculations (Gimsa and Wachner [1])

a	b	c	d	ε_i	ε_m	ε_e	σ_i	σ_m	σ_e
7.7 μm	4 μm	1.85 μm	8 nm	50	9.04	80	0.53 S/m	1 μS/m	0.12 S/m

2.2. Deriving characteristic equations

For certain frequency regions, equations 3–7 can be simplified even further (see Ref. [1]). Three frequency ranges can be distinguished [1]. Whereas at low frequencies, the high membrane impedance determines field distribution, in the medium frequency range beyond the membrane dispersion, Z_m^* is determined by the membrane capacitance and can be omitted. In this frequency range, the conductivity contributions of the external and internal media dominate the field distribution. In contrast, at very high frequencies permittivities are dominating. The membrane field at mem1 is of a similar order (or even larger than) the fields of the cytoplasm and the external medium. (See Table 1).

To calculate the frequency and cell shape dependence of the absorption of a hypothetical cell, the full equations and typical parameters were used (Fig. 1) [1].

For the parameters given in Table 2, E_{ext1} reaches its highest value in the medium frequency range.

2.3. Molecular dispersions

The frequency-dependent behavior of the absorption can strongly change when frequency-dependent electrical media properties, based on their inherent, molecular structure, are introduced into the model: $\varepsilon = \varepsilon(\omega)$ and $\sigma = \sigma(\omega)$ [5].

3. Conclusions

Our model describes the local field and absorption distribution inside and around ellipsoidal cells. Two structural dispersion processes, the membrane and the bulk conductivity dispersions, influence the local field distribution in dependence on the field frequency and, consequently, the absorption within the cytoplasm, the membrane and the external medium. We found a strong dependence of the local absorption on the geometrical properties of the model. An advantage of our approach is that the model calculations can be directly tested by dielectric single-cell spectroscopy methods such as dielectrophoresis and electrorotation. Such experiments are essential to obtain more detailed information on nonstructural dispersion processes.

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